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Effect of thermal losses on the microscopic two-step heat conduction model

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Abstract

The effects of radiative and convective thermal losses on the thermal behavior of thin metal films, as described by the microscopic two-step heat conduction model, are investigated. It is found that radiative losses from the electron gas are significant in thin films having $(L/T_e^3) < 10^{-22}$, while radiative losses from the solid lattice are significant when $(L/T_e^3)(T_e^4/T_l^4) < 10^{-22}$. Also, it is found that convective losses from the thin metal film are insignificant in most practical operating conditions.

1. Introduction

High-rate heating of thin metal films is a rapidly emerging area in heat transfer [1–13]. When a thin film is exposed to a very rapid heating process, such that induced by a short-pulse laser, the typical response time for the film is an order of picoseconds which is comparable to the phonon–electron thermal relaxation time. Under these situations, thermal equilibrium between solid lattice and electron gas cannot be assumed and heat transfer in the electron gas and the metal lattice needs to be considered separately. Models describing the non-equilibrium thermal behavior in such cases are called the microscopic two-step models. Two microscopic heat conduction models are available in the literature. The first one is the parabolic two-step model [1–5,8–10] and the second one is the hyperbolic two-step model [1,3,7,11].

Ultrafast heating of metals consists of two major steps of energy transfer which occur simultaneously. In the first step, electrons absorb most of the incident radiation energy and the excited electron gas transmits its energy to the lattice through inelastic electron–phonon scattering process [1,3]. In the second step, the incident radiation absorbed by the metal film diffuses spatially within the film mainly by the electron gas. For typical metals, depending on the degree of electron–phonon coupling, it takes about 0.1–1 ps for electrons and lattice to reach thermal equilibrium. When the ultrafast heating pulse duration is comparable with or less than this thermalization time, electrons and lattice are not in thermal equilibrium. As a result, the thermal behavior of the thin film under the effect of the microscopic parabolic heat conduction model is described by [1,3]

$$C_l(T_l)\frac{\partial T_l}{\partial t} = G(T_e - T_l), \tag{1}$$

$$C_e(T_e)\frac{\partial T_e}{\partial t} = \nabla \cdot (k_e \nabla T_e) - G(T_e - T_l) + Q_e. \tag{2}$$

Using different solving techniques, the microscopic parabolic heat conduction model has been used numerously [4,5,8–10,13] to describe the thermal behavior of thin metal films under different applications, operating conditions, geometrical parameters and metal properties. Most of the available investigations have assumed that heat losses are negligible [1–3,8–10,12]. This is due to two reasons.The first reason is the fact that the duration of the heating process is very short and as a result, the metal film does not have enough time to lose energy to the surrounding. The second reason is the

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Nomenclature

C	heat capacity (J/m^3 K)
C_R	heat capacity ratio (C_l/C_e)
g	total energy flux evolved by the heating source (J/m^2)
G	electron–phonon coupling factor (W/m^3 K)
h_c	convective heat transfer coefficient (W/m^2 K)
H_c	dimensionless convective thermal losses, $2h_c/(LG)$
H_r	dimensionless radiative thermal losses, $2\sigma\epsilon T_\infty^3/(LG)$
k	thermal conductivity (W/m K)
L	film thickness (m)
Q_e	heat source within the electron gas (W/m^3)
s	heat source (W/m^3)
S	dimensionless heat source, $s/(GT_\infty)$
t	time (s)
T	temperature (K)
T_∞	ambient temperature (K)

Greek symbols

ϵ	emissivity
η	dimensionless time, tG/C_e
θ	dimensionless temperature, T/T_∞
θ_0	dimensionless electron gas initial temperature, $T_e(0)/T_\infty$
σ	Stefan–Boltzmann constant

Subscripts

c	convective losses
e	electron gas
l	solid lattice
r	radiative losses

fact that hot electrons exchange energy with cold lattice through a phonon–electron coupling factor G having an order of magnitude of 10^{16} (W/m^3 K) which is much larger than any other thermal exchange coefficient. As a result, hot electron gas prefers to transmit most of its energy to the cold solid lattice instead of the surrounding. However, there are applications in which the intensity of the laser heating source is very high and the metal film is very thin. Under these applications, electron gas attains very high temperatures during the early stages of the heating process. As a result, radiative losses from the film through its very large surface area-to-volume ratio becomes comparable with the rate of energy exchange between electron gas and solid lattice.

Up to the authors knowledge, no qualitative or quantitative descriptions for the conditions under which thermal losses from the film may be neglected exist. The aim of the present work is to investigate the effect of thermal losses on the thermal performance of thin metal under the effect of the parabolic two-step heat conduction model. Also, the effects of operating conditions, geometrical parameters and metal properties on the thermal losses are investigated.

2. Analysis

Consider a very short laser pulse on a pure metal film of thickness L. In the following analysis, the interest is focused on the effect of thermal losses from the thin film. As a result, thermal diffusion may be neglected and the film is considered as a lumped system. This assumption is reasonable since the metal film is very thin and has a very high thermal conductivity. Neglecting the temperature dependence of the thermal properties and assuming the incident radiation to be totally absorbed by the electron gas, the governing equations in dimensionless form are given as

$$\frac{\partial\theta_e}{\partial\eta} = -(\theta_e - \theta_l) - H_{r,e}(\theta_e^4 - 1) + S_e, \tag{3}$$

$$C_R\frac{\partial\theta_l}{\partial\eta} = (\theta_e - \theta_l) - H_{r,l}(\theta_l^4 - 1), \tag{4}$$

where

$$\theta = \frac{T}{T_\infty},\quad \eta = \frac{tG}{C_e},\quad S_e = \frac{s_e}{GT_\infty},\quad H_{r,e} = \frac{2\sigma\epsilon_e T_\infty^3}{LG},\quad H_{r,l} = \frac{2\sigma\epsilon_l T_\infty^3}{LG}.$$

Due to the assumption that the film is lumped in the transverse direction, thermal losses appear as a source term in Eqs. (3) and (4).

Radiation thermal losses from the surface of the thin film become significant when their values are comparable with the rate of energy exchange between electron gas and solid lattice. Assume that radiative losses are significant when the ratio of the radiative losses rate to electron–lattice energy exchange rate is an order of 10% or higher. This implies that

$$\frac{H_{r,e}(\theta_e^4 - 1)}{\theta_e - \theta_l} > 0.1. \tag{5}$$

Radiation thermal losses are significant when θ_e has high values which is true during the heating process and before the end of the thermalization period. During the heating process $\theta_e \gg \theta_l$ and when a strong heating source is used $\theta_e^4 \gg 1$. As a result, criterion (5) is reduced to

$$\frac{L}{T_e^3} < \frac{20\sigma\epsilon_e}{G}. \tag{6}$$

In most practical applications, G is of order of magnitude of 10^{16} (W/m^3 K) [1]. This implies that radiation thermal losses from the film are significant in metal films having

$$\frac{L}{T_e^3} < 10^{-22}. \tag{7}$$

A similar analysis conducted on Eq. (4) reveals that radiation losses from the solid lattice are significant if metal films obey the following criterion:

$$\frac{LT_e}{T_l^4} < 10^{-22} \ (\mathrm{m/K^3}). \tag{8}$$

To examine the effect of convective losses from the surfaces of the film, Eqs. (3) and (4) are rewritten, after including these losses and excluding radiative losses, as

$$\frac{\partial\theta_e}{\partial\eta} = -(\theta_e - \theta_l) - H_{c,e}(\theta_e - 1) \tag{9}$$

$$C_R\frac{\partial\theta_l}{\partial\eta} = (\theta_e - \theta_l) - H_{c,l}(\theta_l - 1), \tag{10}$$

where

$$H_{c,e} = \frac{2h_{c,e}}{LG}, \quad H_{c,l} = \frac{2h_{c,l}}{LG}.$$

A criterion similar to Eq. (7) reveals that convective losses are significant in metal films having

$$\frac{L}{h_{c,e}} < 10^{-15}. \tag{11}$$

Comparing the convective with the radiative losses reveals that convective losses may have the same order of magnitude as the radiation losses if

$$h_{c,e} \sim \sigma T_e^3.$$

In most practical operating conditions, $h_{c,e}$ is of order of magnitude of 10 (W/m^2 K) and T_e is of order of magnitude of 2000 K. Under these operating conditions, it is obvious that $h_{c,e} < \sigma T_e^3$. This implies that neglecting convective losses from the film boundaries is justified.

3. Quantitative analysis

To present a quantitative analysis for the importance of the radiative losses from the film, a modified version of Eqs. (3) and (4) has to be solved numerically. The modified version of Eqs. (3) and (4) assumes that the laser heating source evolves its energy instantaneously at $\eta = 0$ and this energy is absorbed immediately by electron gas. This assumption is justified when the duration of the heating source is much less than the duration of the thermalization process. In the literature, heating process of very short duration is simulated in the form of a Dirac delta function $\delta(\eta)$ which assumes that the incident laser beam evolves all of its energy at time $\eta = 0$ [8]. As a result, the heating source which appears in Eq. (3) may be dropped and its effect is considered through the electron gas initial temperature $T_e(0)$ which is found from

$$T_e(0) = T_\infty + \frac{g}{C_e L},$$

where g is the total energy flux (J/m^2) evolved during the heating process. As a result, Eqs. (3) and (4) are rewritten as

$$\frac{\partial\theta_e}{\partial\eta} = -(\theta_e - \theta_l) - H_{r,e}(\theta_e^4 - 1), \tag{12}$$

$$C_R\frac{\partial\theta_l}{\partial\eta} = (\theta_e - \theta_l) - H_{r,l}(\theta_l^4 - 1) \tag{13}$$

with the following initial conditions:

$$\theta_e(0) = \theta_0, \quad \theta_l(0) = 1. \tag{14}$$

Eqs. (12)–(14) are solved numerically to investigate the metal film thermal behavior under different operating conditions.

Figs. 1 and 2 show the effect of the heat capacity ratio C_R on the transient behavior of the electron gas and solid lattice temperatures. Due to its lower thermal capacity and its direct interaction with the incident radiation, electron gas attains much higher temperatures as compared to the solid lattice. In Figs. 1 and 2, $H_{r,e}(= H_e) = 0.1$ and $H_{r,l}\ (= H_l) = 0.01$, which implies that radiation losses have significant effect on the film thermal behavior. As a result, large fraction of the electron gas energy is lost to the surrounding. The remaining small fraction of this energy is transmitted to the solid lattice. This small fraction is unable to make a significant change in the solid lattice temperature due to its high thermal capacity. As a result, the thermal capacity ratio has insignificant effect on the metal film thermal behavior.

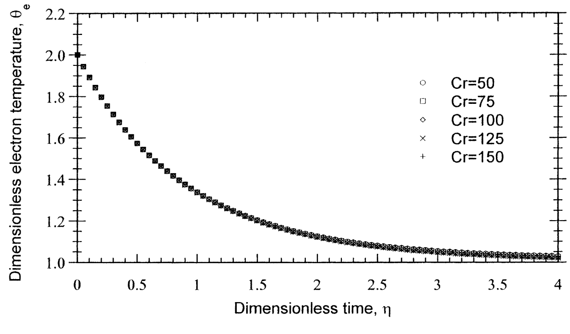

Fig. 1. Effect of thermal capacity ratio on the transient behavior of the electron gas temperature. $H_e = 0.1$, $H_l = 0.01$, $\theta_0 = 2$.

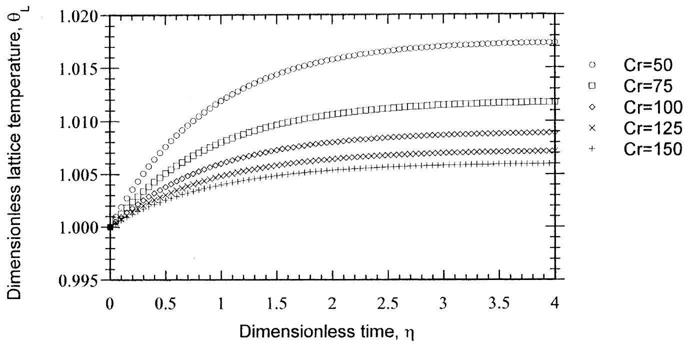

Fig. 2. Effect of thermal capacity ratio on the transient behavior of the solid lattice temperature. $H_e = 0.1,\ H_l = 0.01,\ \theta_0 = 2$.

Fig. 3 shows the effect of the electron gas initial temperature, which is proportional to the intensity of the heating source on the electron gas temperature. As it is clear from this figure, electron gas initial temperature has insignificant effect on the time required by the film to be in thermal equilibrium with ambient conditions. The hotter the film initially is, the faster the cooling rate occurs.

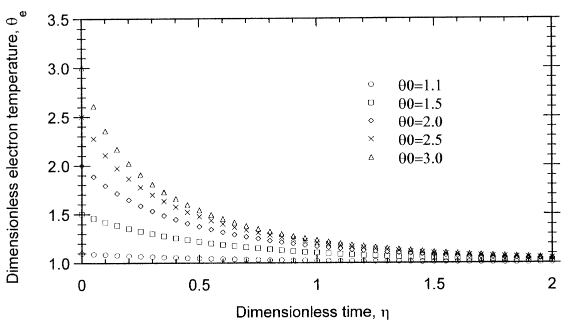

Fig. 3. Effect of intensity of the heating source on the trnasient behavior of the electron gas temperature. $H_e = 0.1,\ H_l = 0.01,\ C_r = 100$.

Fig. 4 shows the effect of the electron gas initial temperature on the transient behavior of the solid lattice temperature. Due to its high thermal capacity, and due to the significant fraction of electron energy lost directly to the surrounding, the solid lattice does not sense appreciably the variation in the electron gas initial temperature.

Fig. 5 shows the effect of the electron gas radiative coefficient $H_{r,e}$ $(= H_e)$, which measures the intensity of the radiative losses from the electron gas, on the percentage reduction in the electron gas temperature as compared to the base value. The electron gas base value is estimated by assuming that radiative thermal losses are absent, i.e., $H_{r,e} = H_{r,l} = 0$. The percentage reduction in the electron gas or solid lattice temperature is defined as

$$\frac{\theta_{\text{without}} - \theta_{\text{with}}}{\theta_{\text{without}}},$$

where θ_{with} is the temperature estimated by including thermal losses and θ_{without} is the temperature estimated

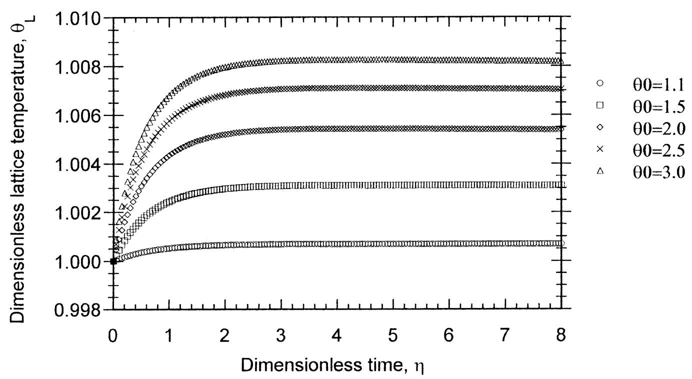

Fig. 4. Effect of intensity of the heating source on the trnasient behavior of the solid lattice temperature. $H_e = 0.1,\ H_l = 0.01,\ C_r = 100$.

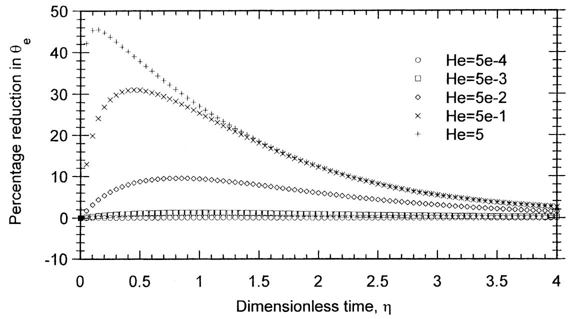

Fig. 5. Effect of electron gas radiation coefficient on the percentage reduction in the electron gas temperature. $H_l = 0.01$, $C_r = 100$, $\theta_0 = 2$.

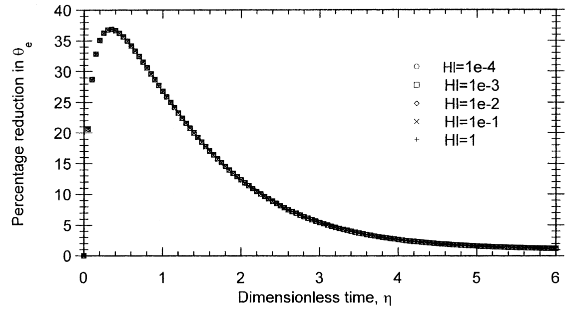

Fig. 7. Effect of the solid lattice radiation coefficient on the percentage reduction in the electron gas temperature. $H_e = 0.01$, $C_r = 100$, $\theta_0 = 2$.

by excluding the thermal losses. It is clear from this figure that radiation losses become significant at radiation coefficients equal to 0.05 or higher. The deviation of the results from the base values is shifted towards the early stages of time as the radiation coefficients increase.

Fig. 6 shows that the deviation in the solid lattice temperature from its base value is insignificant for the same reasons mentioned previously.

Fig. 7 shows the effect of the solid lattice radiative coefficient $H_{r,l}$ $(= H_l)$, which measures the intensity of the radiative losses from the solid lattice, on the deviation in the electron gas temperature. It is clear from this figure that the electron gas temperature is insignificant to the radiative losses from the solid lattice. Due to the solid lattice low temperature, radiation losses from the lattice are insignificant and as a result, their effects on the electron gas thermal behavior are weak. Figs. 8 and 9 show the effect of the electron gas initial temperature on the deviation in the electron gas and solid lattice temperatures, respectively, from their base values. As predicted, the deviation increases significantly as θ_0 increases since radiative losses are proportional to θ_e^4.

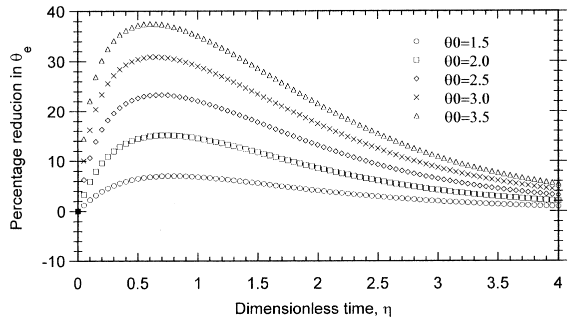

Fig. 8. Effect of the electron gas initial temperature on the transient behavior of the percentage reduction in the electron gas temperature. $H_e = 0.1$, $H_l = 0.01$, $C_r = 100$.

4. Concluding remarks

The effects of thermal losses on the microscopic two-step heat conduction model is investigated. It is found that the radiative losses from the electron gas may have significant effects on the thermal behavior of the thin metal film when $(L/T_e^3) < 10^{-22}$, while radiative

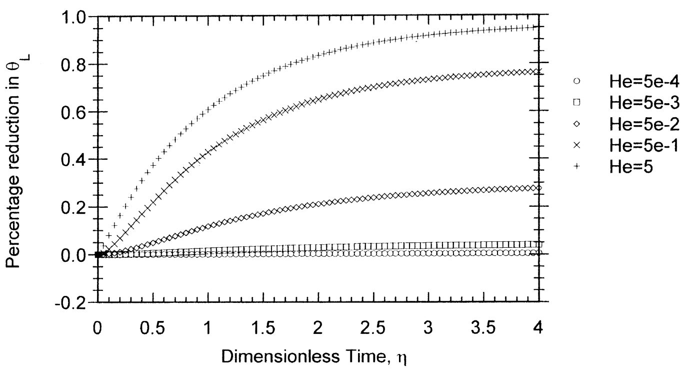

Fig. 6. Effect of electron gas radiation coefficient on the percentage reduction in the solid lattice temperature. $H_l = 0.01$, $C_r = 100$, $\theta_0 = 2$.

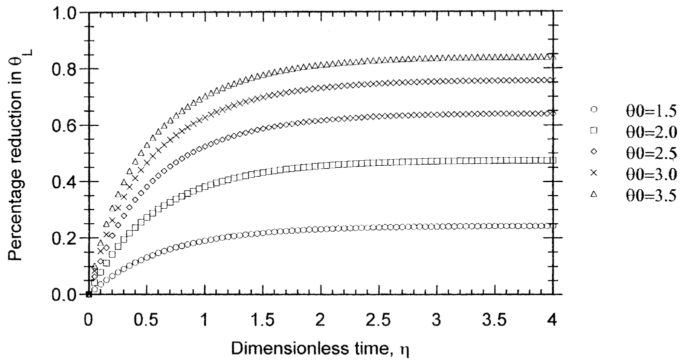

Fig. 9. Effect of the electron gas initial temperature on the transient behavior of the percentage reduction in the solid lattice temperature. $H_e = 0.1,\ H_l = 0.01,\ C_r = 100$.

losses from the solid lattice are significant when $(LT_e/T_l^4) < 10^{-22}$. Also, it is found that convective losses from the thin metal film are insignificant in most practical operating conditions. However, both kinds of thermal losses are significant near the end of the thermalization period when the temperature of the electron gas approaches that of the solid lattice. The thermal capacity ratio has insignificant effect on the film thermal behavior at large radiative heat transfer coefficients.

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